Long-Term Care: Risk Description of a Spanish Portfolio and Economic Analysis of the Timing of Insurance Purchase

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Résumé: Ce papier analyse les motivations à l'achat de l'assurance dépendance, à partir d'une analyse statistique de données d'assurance et de l'utilisation d'un modèle de cycle de vie. Nous faisons un bref survol des arguments en faveur et en défaveur de l'achat d'assurance dépendance. Ensuite des lois d'occurrence de dépendance irréversible et de durée de vie dans cet état sont estimées sur un portefeuille de contrats d'assurance d'une mutuelle espagnole. Des effets calendaires sont estimés, conjointement avec les effets de l'âge et du genre. Ces résultats statistiques sont ensuite inclus dans un modèle de cycle de vie dépargne et d'achat d'assurance. Une application numérique de ce modèle est proposée, qui conduit à un âge d'achat de quarante ans.

Abstract: This paper analyzes the rationale of long-term care insurance purchasing, from a statistical analysis of insurance data and a life cycle model. We make a short survey of the pros and cons of LTC insurance purchase. Then risk distributions in the occurrence and duration dimension are estimated on a Spanish portfolio. Calendar effects are estimated besides age and gender. These statistical results are integrated in a life cycle model of savings and insurance purchasing. A numerical illustration is also provided, which leads to an optimal age of forty years for insurance purchase.

Mots clés : Assurance dépendance. Modèles de Lee-Carter. Approche cycle de vie. Effets calendaires, d'âge et de genre.

Key Words : Long-term care insurance. Lee-Carter models. Life cycle approach. Calendar, gender and age effects.

Classification JEL: C41, D81.

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1 Introduction

The continuous ageing of the population in developed countries creates a need for long-term care services. These services provide assistance to individuals subject to old-age dependency. They are often termed community care and are a substitute to the care-giving traditionally provided by the family and neighbours of the dependent individual. In southern European countries, the role of community care is still incipient as compared to Northern Europe. A 2005 survey shows that Spanish families provide care for 69 percent of the non-institutionalized population.[1] In the United Kingdom, the corresponding percentage is equal to 32. This disparity can be explained by a wider spread of community care in the UK. However in Spain, the number of older people that live alone or with a partial informal support system (either due to the death of a spouse or to a limited number of children) is increasing. Therefore, the risk that they will need assistance or residential services is escalating.

In 2006, the Spanish government instituted a law to promote personal autonomy and care to persons in a situation of dependency (Ley de Promoción de la Autonomía Personal y Atención a las Personas en Situación de Dependencia). This law was designed to function as the "fourth pillar" of the welfare system at all coverage levels, but during 2007, only those having the highest degree of severity received public subsidies. The system is financed by public funding at the state, regional and local level. It is not based on a compulsory contribution as in Germany, and the benefits are first provided on the basis of a severity score. In the case of the highest severity of impairment, eligibility is based only on that severity score. If the level of severity is lower, eligibility is determined also on the basis of income and wealth. Due to the high costs of LTC, it became obvious shortly after the system had been implemented that the overall cost of the coverage was much higher than predicted by the law. One can argue that due to a strong marketing campaign by the Government, most individuals initially believed that the public system would cover all LTC needs, but the Spanish public system is designed as a social right in very acute situations.

In this context, most insurers claim to be prepared to launch LTC products for the Spanish market, but actually only a few of them really offer this type of insurance. The reason put forward is the lack of demand. As a consequence, insurers have insisted that LTC products must be tax qualified in order to be attractive to consumers. The recently approved tax regulation permits a deduction of the premiums paid to LTC insurance for severe and total dependence (up to €12500 per year) from income for fiscal purposes. Unfortunately, at this point, most individuals are confused by the changing regulations and do not know ex-

[1] Costa-Font and Font-Vilalta (2006).

actly what their legal rights are with respect to public LTC coverage and what private insurance can really offer.

There are two polar insurance markets for private LTC insurance. The French market provides fixed indemnity coverage in the form of monthly annuity payments. On the contrary, the American market operates on the basis of reimbursement models. The American model is subject to escalating costs, especially through court decisions and a moral hazard effect.[2] However a reimbursement insurance contract provides a better hedging of risks. The Spanish market is developing along the lines similar to those retained in France. We will implicitly refer to this type of insurance product in risk assessment and in the economic analysis of insurance demand. Hence the components of LTC risk assessed in the paper are the occurrence and the duration of the LTC spell, but not the severity of impairment and its financial consequences. We summarize the pros and cons of LTC insurance purchase in the next section. Then we present a statistical analysis of a Spanish portfolio of permanent disability insurance contracts sold by a mutual insurance company. They include long-term care for the elderly, and we describe both the occurrence risk of an irreversible disability state, and the duration in this state. Our statistical study includes an analysis of calendar effects. In the Section "Life cycle analysis of ex ante LTC insurance demand," we integrate our statistical results into an economic model of optimal insurance purchasing in order to question the timing of LTC insurance purchase during a lifetime. We use an extension of Yaari's life cycle model,[3] with the addition of an irreversible disability state. We present a numerical illustration of the model in which the optimal age of purchase is about forty years. We also perform a sensitivity analysis of the consumer's behaviour with respect to the main parameters of the life cycle model. We discuss the limits of our approach (selection bias issues as insurance demand is assessed from a portfolio of policies, connections with usual insurance demand models). Lastly, we draw conclusions and we clarify our future research directions. Technicalities are relegated to an appendix available at the URL
http://ceco.polytechnique.fr/publications/ (working paper 2008-03).

2 The pros and cons of long-term care insurance purchase

Let us detail the arguments in favour of LTC insurance purchase. Some of them are derived from a risk analysis of LTC. There are two components in LTC risk.

[2] See Kessler (2008) for more developments.
[3] Yaari (1965).

The first component is the occurrence risk of facing an LTC spell during the life cycle, and the second is the duration risk in the LTC state (together with the severity of impairment). A decrease in the occurrence probability of an LTC spell should be followed by a similar variation of the price of LTC insurance, and make insurance purchase less deterred by potential losses due to lapses. Indeed, an LTC contract is a very illiquid asset, which is usually not or partially repaid in case of surrender. The probability of using the coverage on the life cycle strongly depends on the contract type. A fixed indemnity coverage only insures irreversible LTC spells, as the policyholder receives an annuity. The probability of facing an LTC spell during the life cycle is close to ten percent on our data for the representative consumer retained in our life cycle analysis (a man, born in 1950). When insurance contracts follow a reimbursement model, they usually cover reversible dependence risk and the aforementioned probability often exceeds twenty five percent. Because of risk aversion, the willingness to buy LTC coverage increases with the expected duration of LTC and with the heterogeneity of duration distributions. Besides, an overestimation of risks is often observed for low frequencies and increases the motivation to insure.[4] Individuals who engage in more preventive health activity tend to systematically overestimate the probability of a dependence spell.[5] In this respect, long-term care risk could be a source of "advantageous selection", at least for young buyers of insurance. This means that insurance purchase might send a positive signal on the risk level, a result opposite to the usual "adverse selection" effect.[6] Lastly, the motivation to buy LTC coverage increases with bequest motives.

The first argument against the purchase of LTC insurance is the crowding out effect by public insurance. In Germany, for instance, a fifth pillar of the social security system was created in 1995 and it leaves little room for private insurance.[7] A high loading factor for LTC insurance may also deter potential buyers. An statistical study of American contracts quotes an average value of 0.18 if the policy is held until death, whereas the loading factor raises to 0.51 if we account for lapses.[8] The presence of children can work in two ways. On one hand, it creates bequest motives and a motivation to buy insurance coverage. On the other hand, the elderly may fear that, if they purchase insurance, children may institutionalize them when they are unable to act on their own. Therefore,

[4] Kunreuther et al. (1978).

[5] Finkelstein and McGarry (2006).

[6] See De Meza and Webb (2001) for an economic analysis of "advantageous selection", and Fang, Keane and Silverman (2008) for econometric developments on adverse vs. advantageous selection on the Medigap insurance market in the USA.

[7] See Taleyson (2003) for a survey of the LTC insurance market along with public financing of long-term care in the world.

[8] Brown and Finkelstein (2007).

an elderly person who prefers attention from children over purchased help may decide not to buy long-term care insurance.[9] Another reason not to buy LTC insurance is the lower utility of consumption in a poor health state.[10]

Other problems arise besides the insurance purchasing choice. The first is the optimal date of purchasing. Indeed, there is an irreversibility issue: the insurance contract often cannot be repaid before entry into the LTC state. Hence, there is a timing decision which must be made as in any real options issue. Second, there is an alternative solution to *ex ante* insurance, which consists in buying annuities once the LTC state is reached. Several products of this type are proposed, which are termed immediate annuities, reverse mortgages, etc. The wealth effect on the willingness to buy LTC insurance is not straightforward. On one hand, the deterrence effect of repayment risk should decrease with income and wealth. Besides, bequest motives increase with wealth and motivate the hedging of LTC risk. On the other hand, the motivation to use immediate annuities as a substitute for *ex ante* insurance (i.e., to self-insure against occurrence risk) increases with wealth. For instance, home equity can be sold in return for a life annuity, should an LTC spell occur.[11]

3 Statistical analysis of a Spanish data base

3.1 Presentation of the data base

In Spain some insurers have sold LTC coverage for many years as an extension to permanent disability insurance. Individuals who claim not to be able to perform daily life activities receive an annuity, even if they already retired. The data analyzed here comes from a sample of contracts drawn at random from a mutual insurance company. We have 150000 insurance contracts and 2800 LTC spells in round figures. The information was collected for thirty years, which allows us to study calendar effects from 1975 to 2005. The file contains all the available data for policyholders having underwritten the same product. The product is a disability coverage that provides a monthly compensation when a person is declared disabled. The state of disability is assessed by doctors appointed by the company on the basis of standard medical and physical tests. Disability is equivalent to a severe dependence level, where the individual is not able to perform daily life activities without the assistance of another person. Conditions to become eligible are very strict. In the contract they are defined as "a permanent

[9] See Sloan and Norton (1997) for further developments and an empirical study on the AHEAD and HRS data bases.

[10] Pauly (1990).

[11] Davidoff (2008).

and irreversible loss of the capacity to function autonomously due to: irreversible psychotic disorder, hemiplegia, paraplegia, severe Parkinson disorder, aphasia or Wernicke disorder, or dementia due to cerebral malfunction". In addition, due to the traditional practice in the company, blindness or losing two arms or legs are sufficient conditions to grant compensation.

The insurance product analyzed here can be understood as an LTC insurance which complements a permanent disability coverage. It was very popular in the 60s and 70s, mainly due to the disability coverage and to the weakness of public pensions at that time. A striking result is the age of policyholders at the inception of the insurance contract, which is only 28 years, and it increases with calendar time. An explanation is that the policyholders first hedged permanent disability risk, in a context where the public coverage was very weak. Usually buyers of LTC insurance are middle aged or are young retirees. In France, the age of LTC insurance purchasers decreased from 65 to 61 years on average in a few years. The application of a life cycle approach given later in the paper shows however that rather young adults may rationally opt for LTC coverage.

3.2 Statistical results

3.2.1 Duration distributions of LTC spells

Table 1 presents the estimation of a proportional hazards model on the duration of LTC spells. Covariates are the age at the beginning of the LTC spell, the corresponding date in order to allow for a calendar effect, and the gender. The individuals are the LTC spells, and the dependent variable is the pair duration - event indicator. The event is death (1868 deaths out of 2787 LTC spells), which is the only possible exit from LTC in our setting. Dependent policyholders who are still alive at the date of data gathering generate a censored observation.

Table 1: Proportional hazards model for the duration of LTC

Covariates	Estimated coefficient	Estimated standard deviation
x_1 : Age at the beginning of the LTC spell	0.0558	0.00248
x_2 : Date at the beginning of the LTC spell	0.0788	0.00499
x_3 : Indicator of female gender	-0.325	0.166

The proportional hazards model estimates a death rate for individuals in LTC, where the specification of the instantaneous death rate of the policyholder i is

$$\lambda_i(t) = \exp(\alpha_1 (x_1)_i + \alpha_2 (x_2)_i + \alpha_3 (x_3)_i) \times h(t).$$

The duration t is the seniority of the policyholder in the LTC state.

Ceteris paribus, a supplementary year of age increases the mortality rate of 5.7%, and a supplementary calendar year is associated to an 8.2% increase.[12] This second result denotes a decreasing trend in the duration of LTC. Women also stay longer than men in LTC. The estimated instantaneous death rate for the average characteristics ($\overline{x_1} = 70.8$ years; $\overline{x_2} = 1997.4$; $\overline{x_3} = 5.4\%$) is roughly equal to 0.13, with a slightly increasing shape as a function of seniority. This means that the death rate per year in this case is close to 0.12. This death rate is the average for 84-year-old men in Spain in 1997, and for 88-year-old women, regardless of their health state. Entering in an LTC state strongly increases mortality for the elderly, but the relative variation decreases with age and with the seniority in LTC.

3.2.2 Occurrence risk of entry in an LTC spell

The purpose of this section is to analyze the occurrence rate of entry in an LTC spell, depending on the age and calendar time. We did not introduce gender because women represent only five percent of LTC spells and their sample is too small for a regression-based estimation. The relative risk exposure associated to women is equal to twenty percent if measured by the duration, and to seven percent if the age and calendar time are controlled for with the model estimated hereafter. About 150000 policies are considered in the analysis, but there are almost 60000 lapses in the sample. The portfolio reached a peak of 90000 policies in 1992, then it was closed because LTC and permanent disability insurance contracts were redefined for new business. There were 55000 policies remaining in 2005.

The number of entries in LTC for a given age and calendar year is compared to the risk exposure. As we have many zeroes for these numbers but also for risk exposure, we did not perform a direct analysis of the entry rates, as in a Lee-Carter approach. We retained a binomial model and a specification of the hazard rate which is that of Lee-Carter. Let $ne_{t,x}$ be the number of entries in LTC during the year t at the age x, where t and x have integer values. If $ngh_{t,x}$ is the corresponding number of policyholders in good health (i.e. the risk exposure), the binomial model is the following

$$NE_{t,x} \sim B(ngh_{t,x},\, p_{t,x});\; p_{t,x} = 1 - \exp(-\lambda_{t,x});\; \lambda_{t,x} = \exp(a_x + c_t u_x). \qquad (1)$$

The vectors a, c and u represent respectively an average age effect, a calendar effect and a multiplier applied to the calendar effect. For instance, consider two

[12] This calendar effect is too high to be used for plausible long run predictions. Taking into account sub-intervals of calendar time in the estimation leads to rather chaotic results.

different calendar dates t_1 and t_2, and a given age x. The ratio between the corresponding hazard rates is equal to

$$\frac{\lambda_{t_2,x}}{\lambda_{t_1,x}} = \exp((c_{t_2} - c_{t_1}) \times u_x).$$

The last expression leads to an interpretation of u as a multiplier on calendar effects.[13]

Identifying constraints are required for c and u. In a Lee-Carter approach, the principal components c are centered and the norm of the principal factor u is equal to one. We retained these constraints here, with weights proportional to the number of entries.[14] The model needs strictly positive margins on the number of entries in order to be estimated. We restricted the calendar time interval from year 1975 to 2005, and age from 30 to 95 years. The first seven calendar years are gathered in a single observation, due to low values for the number of entries in LTC spells at the beginning. The age levels are three year intervals. Hence we have 25 calendar levels and 22 age levels in the estimation. The estimation of the average effect a is given in Figure 1. The growth rate with respect to age of the entry rate into LTC is almost constant and close to ten percent. This result is close to what is usually observed for death rates (growth equal to 8-9 percent per year for most of the life cycle). The ratio between the entry rate into LTC and the death rate is close to 0.1 for the consumer retained in Section 4 (a man, born in 1950), and the same result holds for the probability of facing an LTC spell before dying.

The average entry rate into LTC increases quickly with calendar time in the portfolio, but this growth partly disappears if age is controlled for, as in this model. Figure 2 exhibits almost constant values for c_t during the last ten years. The coherence with the preceding result is due to the constant ageing of policyholders in the portfolio. There is however a sharp increase in the entry rate in 1992 and 1993. The insurance company modified its rating structure at that time, and this calendar effect could reflect a simultaneous slackening of acceptance rules for LTC spells.

The multiplier u increases beyond sixty years of age (see Figure 3). The increase of the entry rates in LTC observed before 1995 in Figure 2 is only effective beyond sixty years.

[13] The Lee-Carter (1992) model performs a Principal Component Analysis (PCA) on the logarithms of entry rates, which are most often death rates. The vectors a, c and u are respectively the averages of log-rates by age, the principal components and the principal factors of the PCA.

[14] This choice was made from a local expansion of the log-likelihood of the binomial model, interpreted as an euclidean distance between theoretical and empirical frequencies. Anyway, the estimated hazard function does not depend on the identifying constraints.

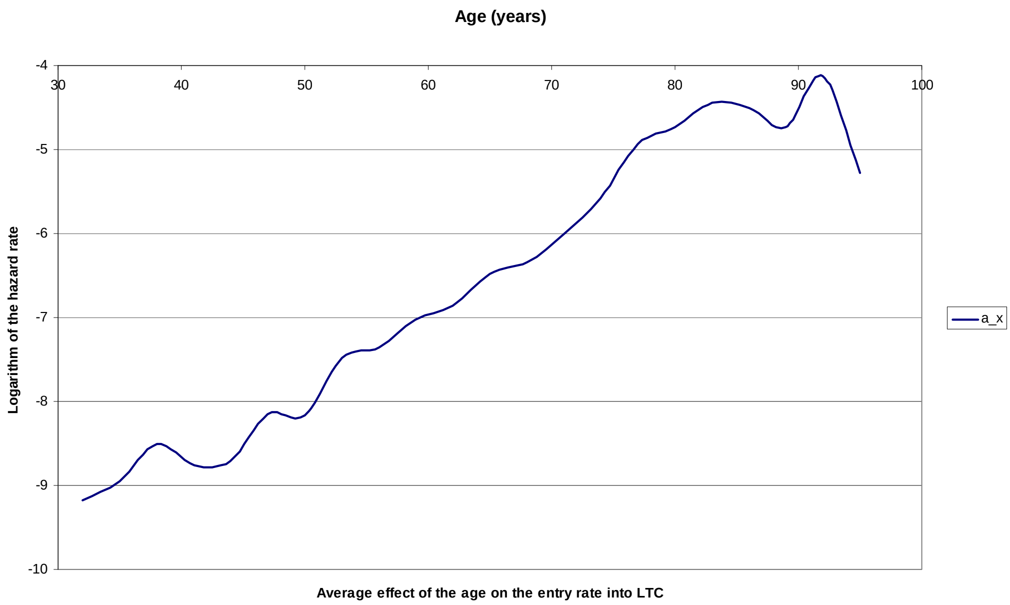
Age (years)
30
40
50
60
70
80
90
100
-4
-5
-6
-7
-8
-9
-10
Logarithm of the hazard rate
a_x
Average effect of the age on the entry rate into LTC

Figure 1:

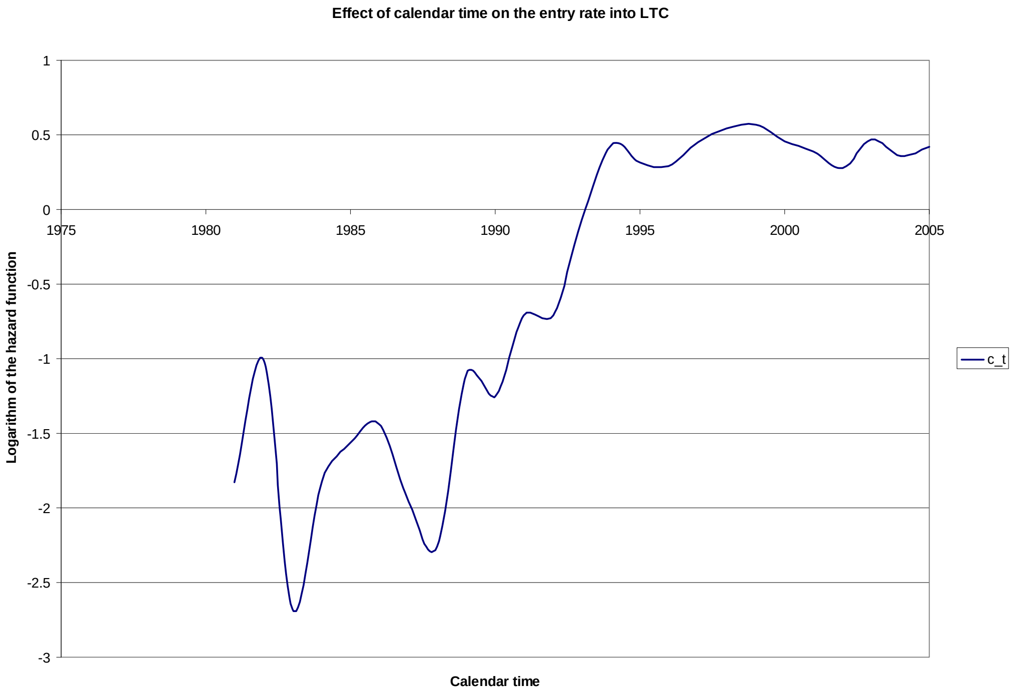
Effect of calendar time on the entry rate into LTC
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
1975
1980
1985
1990
1995
2000
2005
Logarithm of the hazard function
Calendar time
c_t

Figure 2:

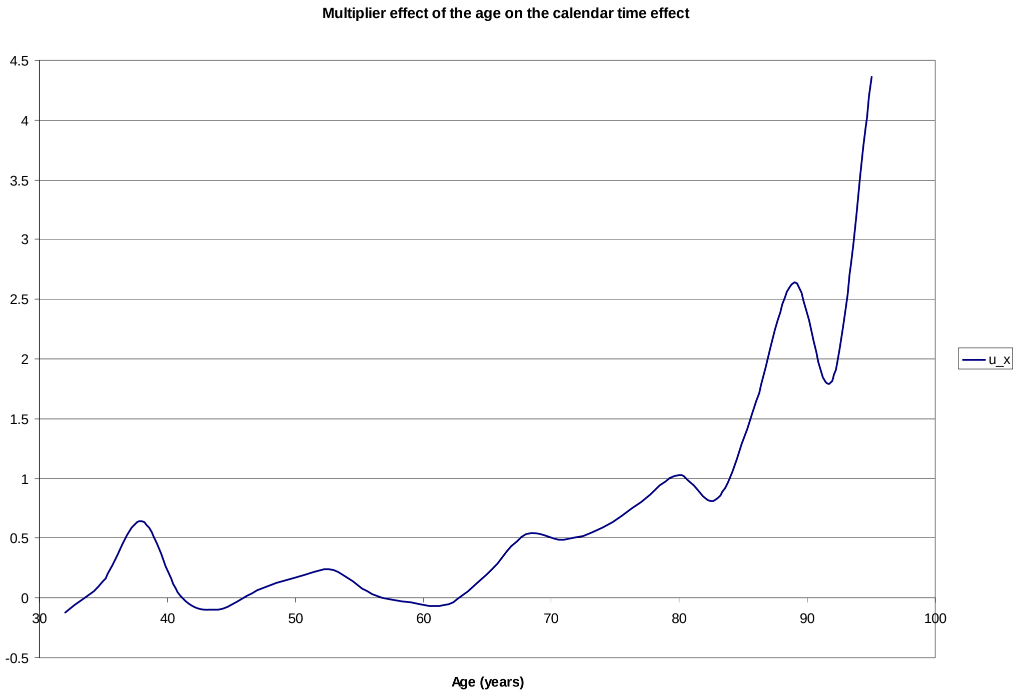
Multiplier effect of the age on the calendar time effect
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
30
40
50
60
70
80
90
100
Age (years)
u_x

Figure 3:

4 Life cycle analysis of ex ante LTC insurance demand

A life cycle analysis of the consumer is the suitable framework to analyze the timing of LTC insurance purchase. In this section, we present an extension of Yaari's model. Yaari's paper is seminal for the life cycle analysis of consumption and savings with life duration risk. It complements the fixed horizon models, where randomness is considered at the income and asset return levels.[15] The rationale for LTC insurance purchase can also be studied in the Merton-Samuelson framework. Gupta and Li[16] follow this approach (the planning horizon is retirement), with a stochastic model on the health capital and also departures from the expected utility model. Our model starts from Yaari's specification (see the appendix for a summary), and includes LTC insurance bought *ex ante* in a model where irreversible dependency is a possible transition between a good health state and death. Adverse selection is low for young buyers of LTC insurance (for which we might even have an "advantageous selection" effect), as discussed in Section 2. Our approach, which applies on LTC insurance demand statistical results which are derived from LTC policies should not create an important selection bias. Using the statistical estimations of the preceding section, we will give a numerical example of optimal insurance purchase and savings behaviour in this context.

Figure 4 shows the life cycle evolution of a consumer's optimal consumption and wealth under the following assumptions.

- The income flow is also given in Figure 4. It increases from 25 to 50 years, then decreases until the age of retirement (65 years). The retirement income is constant and equal to sixty percent of the maximum wage. The wealth at age 25 is equal to zero.

- The utility of consumption u is a CRRA function. The relative risk aversion coefficient is equal to 2, and we retained $u(c) = -1/c$. The CRRA utility function is retained first because the marginal propensity to consume is infinite in zero, which rules out a null consumption at the optimum. In that case, the life cycle model is restricted to the time interval where income is strictly positive. This result simplifies the interpretation of the Euler equation on the optimal consumption path. The Euler equation is fulfilled when positivity constraints on wealth and consumption are not binding. With a CRRA function, this means that wealth is greater than zero. Economic

[15] Merton (1969) and Samuelson (1969).
[16] Gupta and Li (2007).

studies state that a relative risk aversion coefficient should range between 1 and 4 for most of the consumers.[17]

- The utility of bequests (depending on the date of death and on the wealth bequeathed) is obtained from a hump-shaped function of time and from an increasing and concave function of wealth. A plausible optimal wealth path implies that the marginal utility of consumption exceeds the marginal utility of bequests if the wealth is large enough (see Section A.1).

- The real interest rate is equal to 2%, and the subjective discount rate is equal to 3%.

- Lastly, the mortality rates are estimated and predicted from Spanish data at the national level. We estimated a Lee-Carter model on mortality rates drawn from the Human Mortality Database (available at the URL `http://www.mortality.org/`). Mortality rates for individuals in good health are then estimated for each gender in order to make the national mortality rates consistent with our statistical results on LTC. The consumer is assumed to be a man, born in 1950.

From Figure 4, we see that the consumer begins to save at age 41. The wealth reaches a peak at the age of retirement, and then decreases until age 84. The downturn of wealth at the age of retirement is due to the continuity of the optimal consumption path. Hence the decrease in savings compensates the drop in income at that date. At the end of the life cycle, there is no bequest left to the heirs. The last result is unavoidable in the Yaari's model under fairly general conditions (see Appendix A.1).[18]

Usual LTC insurance is bought *ex ante*, i.e. when the policyholder is in good health. The policy covers both an occurrence and a duration risk. Other LTC insurance products are immediate annuities which are bought when the policyholder has reached the LTC state. In that case only the duration risk is covered.

Let us include *ex ante* LTC insurance in this life cycle model. We suppose that the coverage can be bought only once in a lifetime, and that the insurance contract cannot be repaid. Three states of health are defined (good health, irreversible dependency, death). The purchase occurs only in the good health state, and the premium is paid continuously until this state is left. If the policyholder dies at this date, there is nothing left in the bequest from the insurance company. If the policyholder enters the LTC state, he then receives an annuity.

[17] Barsky et al (1997), Gollier (2001).

[18] The original proof is due to Leung (1994).

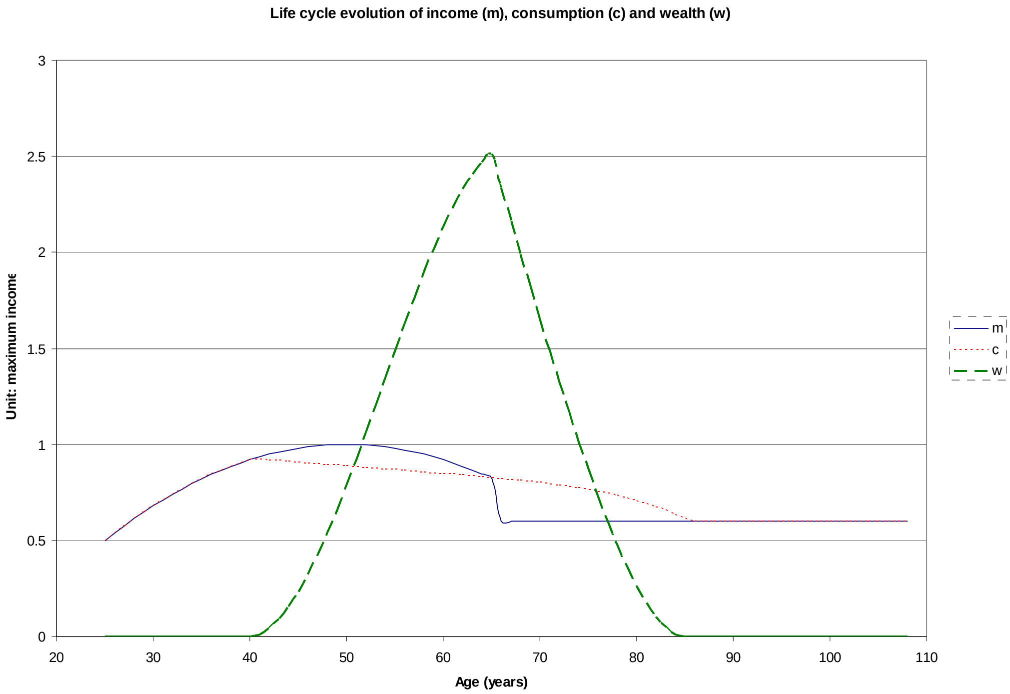
Life cycle evolution of income (m), consumption (c) and wealth (w)
Unit: maximum income
Age (years)
m
c
w
0
0.5
1
1.5
2
2.5
3
20
30
40
50
60
70
80
90
100
110

Figure 4:

The theoretical properties of this model are given in the appendix, when the utility of consumption is the same in the LTC state as in good health. The Euler equation on consumption is not modified by the addition of LTC risk and insurance in Yaari's model if the utility function in the LTC state is the same as in good health. However this equation holds only when the positivity constraint on wealth is not binding. As the income equations are different in the two models, the link between consumption and savings is also different. Lastly, the optimal timing of LTC insurance purchase is given by a "smooth pasting" condition on the residual expected utility of consumption and bequests. This means that a rational consumer should buy the insurance coverage at a date when the marginal value of waiting to purchase is equal to the marginal time value after the insurance purchase.

As an example, let us apply our life cycle model with a cost of LTC free of public benefits which is equal to two thirds of the retirement income. We also assume that the insurance policy completely covers this loss, and that the utility of consumption is the same in the LTC state as in good health. The occurrence and duration risks of LTC are derived from the statistical analysis of the preceding section. We suppose that the insurance policy is priced based on the expected loss, with the interest rate used for the consumer's utility, and with a loading factor equal to thirty percent. We also included a three year qualifying period (i.e. the policyholder is insured if the LTC spell begins at least three years after the inception of the contract). With these assumptions, LTC insurance coverage is bought at 39 years, with a premium level equal to 0.6 percent of the maximum income. The saving behaviour is close to what is observed without insurance, with the same ages for the wealth accumulation and decumulation cycle.

We now perform a sensitivity analysis of the consumer's behaviour with respect to the main parameters. These parameters are the subjective discount rate a, the intertemporal elasticity of substitution $\eta_u(c) = -u^{'}(c)/c\,u^{''}(c)$ (inverse of the relative risk aversion for a separable lifetime utility, hence a constant for a CRRA utility function), the real interest rate r, the income flow and the coverage level cl of LTC risk. Parameters are modified one by one, and the main results (interval where savings are positive, maximum wealth, age at insurance purchase) are given for each scenario.

Table 2: Sensitivity analysis of the consumer's behaviour with respect to the main parameters: $a = 3\%$; $\eta_u = 1/2$; $r = 2\%$; $cl = 100\%$

Departure(s) from the basic scenario	Beginning of the saving period	End of the saving period	Maximum wealth	Age at purchase
None	41	84	2.52	39
$a = 4\%$	45	81	1.65	42
$\eta_u = 1/3$	40	87	2.79	37
$r = 3\%$	38	91	3.86	36
other income flow	47	85	2.39	44
$cl = 50\%$	41	84	2.60	37

Let us briefly comment these results. The maximum wealth is always reached at retirement, due to the drop in income at that date. The saving and insurance purchase decision are deferred if the subjective discount rate increases from 3 to 4%, which is not surprising because preference for present increases then. The saving level strongly decreases, and the Yaari's model cannot match real-world behavior for large values of the subjective discount rate. If relative risk aversion increases from 2 to 3 (hence η_u decreases from 1/2 to 1/3), the saving period is wider and the purchase date decreases. The optimal consumption flow must be flatter since the intertemporal elasticity of substitution decreases, which explains the modification of the saving period. An increase in the real interest rate entails a modification of the output at the opposite of that observed after an increase in the subjective discount rate. In the latter case, the savings modification is similar whereas in the former case the increase in wealth is partly due to the modification of the rate of return on assets. The alternative income flow increases until retirement. The saving decision is delayed because highest income levels are reached later in the life cycle. Lastly, a partial insurance coverage does not modify the saving period, but increases slightly the saving level. Savings can be seen here as a substitute for insurance. Insurance purchase always precedes the beginning of the saving cycle in our examples, and the lag ranges between two and four years. Interpreting this result is not easy because the "smooth pasting" condition which determines the optimal purchase date is based on time derivatives of the residual lifetime utility. The meaning of those items is not easy to grasp.

Let us compare this life cycle approach with usual models of insurance demand. In our model, self-insurance means that the purchase decision is postponed until the end of the life cycle. A pending question is whether self-insurance with respect to LTC occurrence risk together with a saving cycle (very frequent in the real-world) could be derived from this life cycle model. This might not be true, but assumptions of the Yaari's model favour an early purchase of LTC insurance.

For instance, the income flow and the asset return are deterministic in Yaari's model and so is the optimal wealth path. In a risky environment, the consumer would value information on his wealth perspectives and postpone the purchase decision.

5 Conclusions

This paper presented the motivations for LTC insurance purchase. Occurrence and duration risks were estimated on a Spanish portfolio of permanent disability insurance contracts, and we also used national statistics for death rates. Then we presented an economic model of optimal insurance purchasing in order to question the timing of LTC insurance purchase during a lifetime. The life cycle model deals with the transition and duration risks associated to the health states. We restricted the interpretation to a numerical application of the model and to a sensitivity analysis, but we plan to develop later an analysis of the purchase date as a function of the various parameters. We also intend to include *ex post* LTC insurance in the model. The purpose is to assess to what extent it could be a substitute to the classical insurance.

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A Appendix

A.1 The Yaari's model

The model maximizes a lifetime utility under life duration risk, with a bequest motive. Only duration risk is dealt with, and the income flow m is deterministic. The time index is equal to zero at the consumer's birth, and the current date is also the age of the consumer. The expected lifetime utility derived from a consumption flow c, a psychological discount factor α and a utility function of consumption u is equal to

$$E\left[\int_0^T \alpha(t)\, u[c(t)]\, dt\right] = \int_0^{\overline{T}} S(t)\, \alpha(t)\, u[c(t)]\, dt,$$

where T is the life duration, $\overline{T}$ its maximum value and where S is the survival function of the consumer. The consumption is expressed in real terms. The wealth is invested in risk free assets, and the real interest rate is equal to r. If w is the wealth of the consumer, the income splits into consumption and savings from the equation

$$rw + m = c + w^{'}. \tag{2}$$

A utility of bequest is added to the lifetime utility, and is expressed as $\alpha(T)\, \beta(T)\, \varphi(w(T))$. The function φ denotes a utility depending on the amount bequeathed which also integrates possible taxes. We suppose that u and φ are increasing and concave. The utility also depends on the date of death through the function β. We expect β to be hump-shaped, because the importance of bequests is foremost when the consumer dies in his middle years. Then the aggregated expected utility is equal to

$$E\left[\int_0^T \alpha(t)\, u[c(t)]\, dt + \alpha(T)\, \beta(T)\, \varphi(w(T))\right] = \int_0^{\overline{T}} S\, \alpha\, u(c) + \pi\alpha\beta\varphi(w). \tag{3}$$

We dropped the time index in the last integral. We denoted the density of the date of death as π, with $\pi = -S^{'}$. The optimal consumption plan during the whole life maximizes the expected lifetime utility given in (3). The main assumption if that the functions α, β, r, m and S are deterministic. Hence there is risk, but no uncertainty on the life duration. The control variable is the consumption flow and positivity constraints are retained on c and w. The initial wealth is assumed positive, and a positive wealth during the whole life implies that

$$w = 0 \Rightarrow \left(w^{'} \geq 0 \Leftrightarrow c \leq m\right). \tag{4}$$

The last equivalence results from (2). Let us denote the instantaneous mortality rate and the psychological discount rate as $h = -S^{'}/S$; $a = -\alpha^{'}/\alpha$. If the positivity constraints on wealth and consumption are not binding, the optimal consumption flow follows the Euler equations

$$\frac{c^{'}}{c} = \eta_u(c)\ \left[(r-a) + \left(h\left[\frac{\beta\,\varphi^{'}(w)}{u^{'}(c)} - 1\right]\right)\right],\ \eta_u(c) = \frac{u^{'}(c)}{-c\,u^{''}(c)}. \quad (5)$$

The coefficient $\eta_u(c)$ is the intertemporal elasticity of substitution and the inverse of the relative risk aversion. The difference $r - a$ is usually negative, as the subjective discount rate is most often greater than the real interest rate. The consumption growth rate decreases with the mortality rate if $u^{'}(c) > \beta\varphi^{'}(w)$, i.e. if the marginal utility of consumption exceeds the marginal utility of bequests.

The Euler equation constrains the shape of the optimal wealth path on the life cycle. Leung (1994) proves that the optimal wealth is equal to zero at the end of the life cycle in the absence of bequest motives, under fairly general conditions. The sketch of the proof is the following: suppose that the functions r, a and η_u are bounded, and that $u^{'}(0) = +\infty$. The last two conditions are fulfilled for CRRA utility functions, and the last condition implies that the consumption is always greater than zero at the optimum. In that case, the life cycle model is restricted to the time interval where income is strictly positive, unless a positive initial wealth can be considered. Since optimal consumption paths are continuous, we obtain

$$\min_{0\leq t\leq\overline{T}} c(t) = \underline{c} > 0. \quad (6)$$

Suppose that optimal wealth is greater than zero until the end of the life cycle, in which case the Euler equation is fulfilled in the left neighborhood of $\overline{T}$. If the death rate h is increasing, we have that

$$S(\overline{T}) = \exp\left[-\int_0^{\overline{T}} h(t)\,dt\right] = 0 \Rightarrow \lim_{t\longrightarrow(\overline{T})^{-}} h(t) = +\infty. \quad (7)$$

If there is no bequest motive ($\varphi \equiv 0$), the Euler equation and equation (7) yield the inequality

$$\frac{c^{'}}{c} = \eta_u(c)\ \left[(r-a-h)\right] \leq \underline{\eta}\ \left[(\overline{r} - \underline{a} - h)\right]$$

in the left neighborhood of $\overline{T}$. We denote upper bounds with overlines and lower bounds with underlines. If t_1, t_2 $(t_1 < t_2)$ are in this neighborhood, we obtain

$$\log c(t_2) - \log c(t_1) \leq \underline{\eta}\ \left[\left((\overline{r} - \underline{a})(t_2 - t_1) - \int_{t_1}^{t_2} h(t)dt\right)\right]$$

$$\Rightarrow \frac{c(t_2)}{c(t_1)} \leq \exp\left[\underline{\eta}(\overline{r} - \underline{a})(t_2 - t_1)\right] \left[\frac{S(t_2)}{S(t_1)}\right]^{\underline{\eta}}.$$

As $S(\overline{T}) = 0$, we obtain $\lim_{t \longrightarrow (\overline{T})^-} c(t) = 0$ if the Euler equation is fulfilled in the left neighborhood of $\overline{T}$. As it is impossible from (6), the Euler equation is not fulfilled at the end of the life cycle, which means that optimal wealth is then equal to zero.

Leung's argument can be extended in the presence of bequest motives with some supplementary assumptions. Since we have

$$\frac{\beta\, \varphi^{'}(w)}{u^{'}(c)} \leq \frac{\overline{\beta}\, \varphi^{'}(0)}{u^{'}(c)}$$

from the concavity of φ, the inequality

$$\frac{c^{'}}{c} \leq \eta_u(c)\ \left[(r-a) + (h\,(\,x-1))\right],\ x \in]0,1[$$

holds if $c \leq c_x$, with $u^{'}(c_x) = \overline{\beta}\, \varphi^{'}(0)/x$. If c_x is large enough so that optimal consumption paths are below this value, Leung's argument is still valid.

A.2 The Yaari's model with LTC insurance

Yaari's paper includes immediate annuities in the preceding model. The main result is that, if immediate annuities are sold at the actuarial price,[19] the optimal consumption plan is the same as what would be obtained without risk on the life duration (i.e. with a fixed horizon as in the Merton-Samuelson model).

In our model, we define three health states for the consumer: good health (good enough not to necessitate LTC), LTC and death. They are respectively numbered 1,2 and 3. Transitions exist only from state 1 to states 2 and 3 and from state 2 to state 3. Hence the consumer cannot recover good health once he has reached the LTC state.

Let us include LTC insurance in the model. We suppose that the coverage can be bought only once in a lifetime. The purchase can occur only in state 1, and the premium q is paid continuously until state 1 is left. If the policyholder dies at this date, there is nothing left in the bequest from the insurance company. If the policyholder enters the LTC state, an annuity Q is paid continuously by

[19] This means that the rate of return of the immediate annuities is equal to the interest rate - (which assumes that the insurer's assets pledged for these liabilities are risk free), plus the mortality rate. Another important assumption in Yaari's model is that annuities can be sold by the consumer at the purchase price.

the insurance company. Besides, the cost of LTC for the consumer (free of public benefits) is equal to $cltc$. We suppose that $Q \leq cltc < m$, in order to rule out both overinsurance and insolvency. The premium q depends on the date of purchase for obvious actuarial equity reasons.

In our model, the control variables are the consumption flow c and the date of purchase x. A non purchase of LTC insurance can be associated with $x = \overline{T}$. The income equation depends on the state of health and on the purchase indicator. In state 1, we have $m + rw - q(x, \bullet)1_{[x,\overline{T}]} = c + w^{'}$. In state 2 with LTC coverage, the income equation is $m + rw + Q - cltc = c + w^{'}$. If the consumer is uninsured in the LTC state, the equation becomes $m + rw - cltc = c + w^{'}$.

It is easily seen that the Euler equation on consumption is the same as that given in (5) without LTC risk and the related insurance if the utility function in the LTC state is the same that in good health. Let us denote $p_i(x)$ as the probability of being in health state i at the date x, and $p_{i,j}(x,t)$ as the probability of state i at the date x and of state j at the date t. All these probabilities are derived at the birth of the consumer. For instance, the link between the annuity and the premium if the date of insurance purchase is equal to x is given by

$$\int_x^{\overline{T}} p_1\, q(x,\bullet)\, B(x,\bullet) = (1+\tau) \int_{x+D}^{\overline{T}} p_{1,2}(x+D,\bullet)\, Q\, B(x,\bullet),$$

where a) τ is the loading factor derived from the real interest rate; b) D is the duration of the qualifying period; c) $B(x,t) = \exp\left[-\int_x^t r(u)du\right]$ is the discount factor.

We have that

$$t \leq x \Rightarrow p_{1,2}(x,t) = 0;\ p_{2,2}(x,t) = p_2(t).$$

$$t > x \Rightarrow p_{1,2}(x,t) + p_{2,2}(x,t) = p_2(t).$$

If the date of insurance purchase is equal to x, the expected lifetime utility is equal to

$$\int_0^{\overline{T}} p_1\, \alpha\, u(m + rw - w^{'} - q(x,\bullet)1_{[x,\overline{T}]}) + \int_0^{\overline{T}} p_{1,2}(x,\bullet)\, \alpha\, u(m + rw + Q - w^{'} - cltc)$$

$$+ \int_0^{\overline{T}} p_{2,2}(x,\bullet)\, \alpha\, u(m + rw - w^{'} - cltc) + \int_0^{\overline{T}} \pi\alpha\, \beta\, \varphi(w) = \int_0^{\overline{T}} F(\bullet, w, w^{'}).$$

For sake of simplicity, we suppose here that there is no qualifying period. The Euler equation on the wealth w :

$$\frac{\partial}{\partial w} F(\bullet, w, w^{'}) = \left[\frac{\partial}{\partial w^{'}} F(\bullet, w, w^{'})\right]^{'}$$

is expressed as follows

$$\alpha\beta\pi\varphi^{'}(w) + \alpha r u^{'}(c)\left(p_1 + p_{1,2}(x,\bullet) + p_{2,2}(x,\bullet)\right)$$

$$= -\left[\alpha u^{'}(c)\left(p_1 + p_{1,2}(x,\bullet) + p_{2,2}(x,\bullet)\right)\right]^{'}.$$

From

$$p_1 + p_{1,2}(x,\bullet) + p_{2,2}(x,\bullet) = p_1 + p_2 = S,$$

we obtain

$$\alpha\beta\pi\varphi^{'}(w) + \alpha r S u^{'}(c) = -\left[\alpha S u^{'}(c)\right]^{'},$$

and the Euler equation already given in (5). However the income equations given in this section are different from that given in the basic Yaari's model, and the link between consumption and savings is also different.

The optimal consumption flow and the date of purchase were obtained by numerical integration, as the Euler equation is insufficient by itself (it is not fulfilled when optimal wealth is equal to zero) to derive the solution. The residual lifetime utility is derived by backward induction, and the solutions are the control variables which depend on the state variables (the wealth, the health state, the purchase indicator and the date of purchase, the insurance indicator and the entry date in LTC if the consumer is dependent).

We explicit the backward induction equations. Let us denote $V^i_{SV}(t)$ the residual lifetime utility at the date t (supposed to be an integer number of years in what follows), where i is the health state ($i = 1, 2, 3$), and where SV are the other state variables. They are listed below.

- If $i = 3$ (death between $t-1$ and t), we have $SV = w$, and $V^3_w(t) = \beta(t)\varphi(w)$.
- If $i = 2$ (LTC state at the date t), we have $SV = w, j, t_e$, with j the indicator of LTC coverage and t_e the date of entry into LTC. Let us denote $p_{2\to3}(t \,|\, t_e)$ the death probability between t and $t+1$, given that the entry into LTC was first observed at t_e ($t_e \le t$). This probability also depends on gender and on calendar effects, and is derived from the statistical analysis of Section 3. The backward induction equation is

$$V^2_{w,j,t_e}(t) = \max_{c\ge0} u(c)$$

$$+\frac{1}{1+a_t}\left[\left((1-p_{2\to3}(t \,|\, t_e)) \times V^2_{w^+,j,t_e}(t+1)\right) + \left(p_{2\to3}(t \,|\, t_e) \times V^3_{w^+}(t+1)\right)\right],$$

with

$$w^+ = (w+I)\times(1+r_t);\; I = m_t + Q_t 1_{[j=1]} - cltc_t - c.$$

- If $i = 1$ (good health at the date t), we have $SV = w, j, x$ with j the indicator of insurance purchase and x the purchase date if $j = 1$. Let us denote $p_{1\to 2}(t)$ (resp. $p_{1\to 3}(t)$) the transition probabilities between good health and LTC (resp. between good health and death) during $[t,\, t+1]$. If $j = 1$, the backward induction equation is

$$V^1_{w,1,x}(t) = \max_{c\geq 0} u(c) + \frac{1}{1+a_t}\left[\left((1 - p_{1\to 2}(t) - p_{1\to 3}(t)) \times V^1_{w^+,1,x}(t+1)\right)\right]$$

$$+\frac{1}{1+a_t}\left[\left(p_{1\to 2}(t) \times V^2_{w^+,j,t+1}(t+1)\right) + \left(p_{1\to 3}(t) \times V^3_{w^+}(t+1)\right)\right]$$

where the coverage indicator j is obtained from the duration D of the qualifying period by

$$j = 1 \Leftrightarrow t_e = t+1 > x + D \Leftrightarrow t \geq x + D; \text{ else } j = 0.$$

$$w^+ = (w + I) \times (1 + r_t);\ I = m_t - q_{x,t} - c.$$

If LTC insurance is not yet purchased, the purchase indicator becomes a control variable, and we have

$$V^1_{w,0}(t) = \max(f_1\ , f_2),$$

with

$$\text{insurance not purchased: } f_1 = \max_{c\geq 0} u(c)$$

$$+\frac{1}{1+a_t}\left[\left((1 - p_{1\to 2}(t) - p_{1\to 3}(t)) \times V^1_{w^+,0}(t+1)\right)\right]$$

$$+\frac{1}{1+a_t}\left[\left(p_{1\to 2}(t) \times V^2_{w^+,0,t+1}(t+1)\right) + \left(p_{1\to 3}(t) \times V^3_{w^+}(t+1)\right)\right].$$

$$w^+ = (w + I) \times (1 + r_t);\ I = m_t - c.$$

$$\text{insurance purchased: } f_2 = \max_{c\geq 0} u(c)$$

$$+\frac{1}{1+a_t}\left[\left((1 - p_{1\to 2}(t) - p_{1\to 3}(t)) \times V^1_{w^+,1,t}(t+1)\right)\right]$$

$$+\frac{1}{1+a_t}\left[\left(p_{1\to 2}(t) \times V^2_{w^+,0,t+1}(t+1)\right) + \left(p_{1\to 3}(t) \times V^3_{w^+}(t+1)\right)\right].$$

$$w^+ = (w + I) \times (1 + r_t);\ I = m_t - q_{t,t} - c.$$

The consumer is not covered by LTC insurance if he buys the insurance the year before entering in the LTC state if the qualifying period is greater than zero, hence the notation $V^2_{w^+,0,t+1}$.